Design Sensitivity Analysis of Welded Strut Joints on Vehicle Chassis Frame

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Keywords Design sensitivity analysis, Direct differentiation method, Differential algebraic equations (DAEs), Backward difference formula (BDF), Finite difference method (FDM).

Abstract

Design sensitivity analysis of a vehicle system is an essential tool for design optimization and trade-off studies. Most optimization algorithms require the derivatives of cost and constraint function with respect to design in order to calculate the next improved design. This paper presents an efficient algorithm application for the design sensitivity analysis, using the direct differentiation method. A mounting area of suspension that welded on chassis frame is analyzed to show the validity and the efficiency of the proposed method.

1. Introduction

The theory of sensitivity functions and its applicability to the parameter identification or estimation problem has been established and used successfully more than a decade. Recently, increasing concern has shown to the sensitivity analysis for the design and modification of mechanical system.

The performance of mechanical system design is strongly affected by the accuracy of the model and the influence of the design parameters. Hence, whenever a physical response is calculated from a
mathematical model there also arises an interest in the sensitivity of that response with respect to parameters of the problems.

This sensitivity information may be used to assess the effect of uncertainties in the mathematical model, to predict the change in response due to a change in parameters, and to optimize a system with the aid of mathematical optimization techniques.

Analytic synthesis methods in designing mechanical systems were proposed by Erdman and his colleges in Ref. 1. Function generation, trajectory generation, and rigid body guidance problems were considered. Though the analytic synthesis methods are powerful in designing a specific mechanism, the numerical methods have been preferred in coping with design of general mechanical systems.  

Numerical optimization has become a routine procedure in designing a structural system. The design optimization method for the structural systems are developed in size, shape, configuration, and topology optimization. In contrast to the structural design area, there exist few general purpose codes that have design optimization capability of mechanical systems. One of the major difficulties is an efficient and reliable analysis of the design sensitivity of a dynamic response due to a design change. As a result, objective of this paper is to develop an efficient and reliable analysis method of the design sensitivity for on welded joints on chassis frame.

Even though the formulations proposed in the previous studies were general, their applications were relatively simple due to complexity of the formulations. The first fully three dimensional applications are demonstrated by Mani in Ref. 7. The velocity transformation method was used to derive the governing equations of design sensitivity. The formulation complexity problem was resolved by using a symbolic language.

Contribution of this research is as follows. The algorithm for design sensitivity calculation is applied for a practical vehicle system that is consisted of many different types of welded joints to demonstrate its validity and efficiency.

2. Macpherson strut Suspension

The Macpherson strut suspension is shown in Fig. 1. Earle S. Macpherson developed a suspension with geometry similar to the unequal-arm front suspensions using a strut configuration. The strut is a telescopic member incorporating damping with the wheel rigidly attached at its lower end, such that the strut maintains the wheel in the camber direction. The upper end is fixed to the chassis, and the lower end is located by linkages which pick up the lateral and longitudinal forces.

![Fig. 1 The MacPherson strut suspension](attachment:image)


3.1 Implicit Numerical Integration of Equations of Motion

The variational form of the equations of motion for a constrained mechanical system is as follows

$$\delta q^T (M \dot{v} - Q + \Phi^T \lambda) = 0$$
where $\delta q$ is the virtual displacement vector in Euclidean space $\mathbb{R}^n$, $\dot{v}$ is the acceleration vector, and $\lambda$ is the Lagrange multiplier vector for joints in $\mathbb{R}^m$. $\Phi$ represents the position level constraint vector in $\mathbb{R}^m$, and the Jacobian is expressed by $\Phi_q \in \mathbb{R}^{m \times n}$ that is assumed to have full row-rank.

The mass matrix $M$ and the force vector $Q$ are defined as follow

$$
M = \text{diag}(M_1, M_2, \cdots, M_{\text{nb}})
Q = (Q_1^T, Q_2^T, \cdots, Q_{\text{nb}}^T)
$$

where $\text{nb}$ denotes the number of bodies. Since $\delta q$ is arbitrary, the equations of motion are obtained as follow.

$$
\begin{align*}
F(q, v, \dot{v}, \lambda) &= M \ddot{v} - Q + \Phi_q^T \lambda = 0 \quad (3.1.a) \\
F(q, v, \dot{v}, \lambda) &= 0 \quad (3.1.b) \\
\Phi(q) &= 0 \quad (3.1.c)
\end{align*}
$$

Successive differentiations of Eq. 3.1.c yield

$$
\begin{align*}
\dot{\Phi}(q, v) &= \Phi_q v - v = 0 \quad (3.2.a) \\
\ddot{\Phi}(q, v, \dot{v}) &= \Phi_q \dot{v} - v = 0 \quad (3.2.b)
\end{align*}
$$

Equations 3.1 and 3.2 comprise a system of overdetermined differential algebraic equations (ODAEs). An algorithm for the backward differentiation formula (BDF) to solve the ODAEs is given in Ref. 1 as follows.

$$
H(x) = \begin{bmatrix}
F(x) \\
\Phi \\
\Phi_q \\
U_0^T \begin{bmatrix} \frac{h}{b_0} & R_1 \end{bmatrix} \\
U_0^T \begin{bmatrix} \frac{h}{b_0} & R_2 \end{bmatrix}
\end{bmatrix} = 0
$$

where $\zeta_1 = \frac{1}{b_0} \sum_{i=1}^{k} b_i v_{n-i}$ and $\zeta_2 = \frac{1}{b_0} \sum_{i=1}^{k} b_i q_{n-i}$ in which $k$ is the order of integration and $b_i$'s are the BDF coefficients. $x = [\lambda^T, \dot{v}^T, \ddot{v}^T, q^T]^T$ and the columns of $U_o \in \mathbb{R}^{m \times (n-m)}$ constitute bases for the parameter space of the position level constraints. $U_o$ is chosen as $[\Phi_q^T, U_o^T]$, the inverse of which exists. Therefore, the parameter space spanned by the columns of $U_o$ and the subspace spanned by the columns of $\Phi_q^T$ constitute the entire $\mathbb{R}^n$ space.

The number of equations and the number of unknowns in Eqs. 3.3 are same, so Eqs. 3.3 can be solved. The Newton’s numerical method can be applied to obtain the solution $x$.

$$
\begin{align*}
H_x \Delta x &= -H \quad (3.4.a) \\
x^{i+1} &= x^i + \Delta x \quad (3.4.b)
\end{align*}
$$

3.2 Implicit Numerical Integration of Equation of Design Sensitivity

A mechanical system consists of bodies, joints, and force elements. Physical properties of these
elements are described by various parameters. The parameters are defined as design variables in this research and are denoted by

\[ b = [ b_1, b_2, \cdots, b_k ]^T \]  \hspace{1cm} (3.5)

Taking the derivative of Eq. 3.3 with respect to the design parameter vector \( \mathbf{b} \) and appending the BDF integration formula yield the following governing equations of design sensitivity\(^{(d)}\)

\[
G(\mathbf{x}, \mathbf{x}_b) = \begin{bmatrix}
\frac{dF}{db} & \frac{\partial F}{\partial \phi} \\
\frac{\partial \phi}{db} & \frac{\partial \phi}{db} \\
\frac{d\phi}{db} & \frac{d\phi}{db} \\
h' U_0^T (R_3)_b & h' U_0^T (R_4) \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
F_q \Delta q_b + F_v \Delta v_b + F_{\phi} \Delta \phi_b + F_{\lambda} \lambda_b + F_{\mathbf{b}} \\
\phi_q \Delta q_b + \phi_v \Delta v_b + \phi_{\lambda} \lambda_b + \phi_{\mathbf{b}} \\
\phi_q \Delta q_b + \phi_v \Delta v_b + \phi_{\lambda} \lambda_b + \phi_{\mathbf{b}} \\
h' U_0^T (h' \Delta v_b - \Delta \phi_b - \Delta \lambda_b) \\
\end{bmatrix} = 0
\]  \hspace{1cm} (3.6)

where \( h' = \frac{h}{b_0} \) and \( \mathbf{x}_b = [ \lambda_b^T, \Delta v_b^T, \phi_b^T, \mathbf{q}_b^T ]^T \).

Equations 3.6 comprises the same number of equations as the unknowns. The Newton's numerical method can be applied to Eqs. 3.6 for the solution \( \mathbf{x}_b \) as

\[
G_{x_b} \Delta \mathbf{x}_b = -G \hspace{1cm} (3.7.a)
\]

\[
\mathbf{x}_b^{i+1} = \mathbf{x}_b^i + \Delta \mathbf{x}_b \hspace{1cm} (3.7.b)
\]

The below subset equations of Eqs. 3.7.a will be considered first,

\[
\Phi_q \Delta q_b + \Phi_v \Delta v_b + \Phi_{\lambda} \Delta \lambda_b + \Phi_{\mathbf{b}} = -\frac{dF}{db} \hspace{1cm} (3.8.a)
\]

\[
h' \Delta \dot{v}_b - h' \Delta v_b + h' R_5(x) = 0 \hspace{1cm} (3.8.c)
\]

\[
h' \Delta v_b - h' \Delta v_b + h' R_4(x) = 0 \hspace{1cm} (3.8.d)
\]

It can be easily shown that any solution \( \Delta \mathbf{x}_b \) satisfying Eqs. 3.8 is also the solution of Eq. 3.7.a. The \( \Delta \mathbf{v}_b \) and \( \Delta \dot{v}_b \) in Eqs. 3.8.c and 3.8.d are obtained in terms of \( \Delta \mathbf{q}_b \) as follows.

\[
\Delta \mathbf{v}_b = \frac{1}{h'} \Delta \mathbf{q}_b - R_4(x) \hspace{1cm} (3.9.a)
\]

\[
\Delta \dot{v}_b = \frac{1}{h'^2} \Delta \mathbf{q}_b - \frac{1}{h'} R_4(x) - R_5(x) \hspace{1cm} (3.9.b)
\]

Substituting Eqs. 3.9 into Eq. 3.8.a and multiplying both sides of Eq. 3.8.a by \( h'^2 \) yields

\[
K^* \Delta \mathbf{q}_b + h'^2 \frac{\partial F}{\partial \phi} \Delta \lambda_b = R_5 \hspace{1cm} (3.10)
\]

where \( F_q = \Phi_q \) is used and \( K^* \) and \( R_5 \) are defined by

\[
K^* = h'^2 F_q + h' F_v + F_{\phi}
\]

\[
R_5 = -h'^2 \frac{dF}{db} + h' (h' F_v + F_{\phi} R_4 + h'^2 F_{\phi} R_5)
\]

Equations 3.10 and 3.8.b can be rewritten in a matrix form as

\[
\begin{bmatrix}
K^* \Phi_q & \Delta \mathbf{q}_b \\
\Phi_q & \Delta \lambda_b \\
\end{bmatrix} = \begin{bmatrix}
R_5 \\
\frac{d\phi}{db}
\end{bmatrix} \hspace{1cm} (3.11)
\]

Equation 3.12 can be solved for \( \Delta \mathbf{q}_b, \Delta \lambda_b \).

To solve \( \Delta \mathbf{v}_b \), Eq. 3.7.a is rewritten as

\[
U_0^T (h' \Delta \mathbf{v}_b) = U_0^T (\Delta \mathbf{q}_b - h' R_4) \hspace{1cm} (3.12)
\]

Without loss of generality \( U_0^T \) can be chosen as \( N^T K^* \) and Eq. 3.12 can be rewritten as follows.
\[ \mathbf{N}^T \mathbf{K}^*(h' \mathbf{A}\mathbf{v}_b) = \mathbf{N}^T \mathbf{K}^*(\mathbf{A}\mathbf{q}_b - h' \mathbf{R}_4) \]  

(3.13)

where the \( \mathbf{N} \) is chosen such that \( \mathbf{N}_q \mathbf{N} = 0 \). Since the \( \mathbf{N} \) is a null space of the \( \mathbf{N}_q \), Eq. 3.13 can be rewritten in a different form as

\[ \mathbf{K}^*(\mathbf{A}\mathbf{v}_b) + \mathbf{v}_b^T \mathbf{r}_1 = \mathbf{K}^*(c_j \mathbf{A}\mathbf{q}_b - \mathbf{R}_4) \]  

(3.14)

Equations 3.14 and 3.7.a can be solved for \( \mathbf{A}\mathbf{v}_b \) by

\[
\begin{bmatrix}
\mathbf{K}^* & \mathbf{v}_b^T \\
\mathbf{N}_q & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{A}\mathbf{v}_b \\
\mathbf{r}_1
\end{bmatrix}
= \begin{bmatrix}
\mathbf{K}^*(c_j \mathbf{A}\mathbf{q}_b - \mathbf{R}_4) \\
- \frac{d}{db} \mathbf{g} - \mathbf{v}_b \mathbf{A}\mathbf{q}_b = \mathbf{v}_b
\end{bmatrix}
\]  

(3.15)

The \( \mathbf{A}\mathbf{v}_b \) can be obtained by taking a similar process to the \( \mathbf{A}\mathbf{v}_b \) as

\[
\begin{bmatrix}
\mathbf{K}^* & \mathbf{v}_b^T \\
\mathbf{N}_q & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{A}\mathbf{v}_b \\
\mathbf{r}_2
\end{bmatrix}
= \begin{bmatrix}
\mathbf{K}^*(c_j \mathbf{A}\mathbf{q}_b - \mathbf{R}_4) \\
\frac{d}{db} \mathbf{g} - \mathbf{v}_b \mathbf{A}\mathbf{q}_b = \mathbf{v}_b
\end{bmatrix}
\]  

(3.16)

The formulation presented above are implemented as in Fig. 2.

**4. Numerical Example**

To show the validity of the proposed formulation, dynamic analysis of a passenger vehicle is performed. The Macpherson strut and multi-link suspensions are employed as its front and rear suspensions. The list of vehicle parameters and their nominal values assumed in analysis is given in Table 1.
Table 1. Vehicle data for sensitivity analysis

<table>
<thead>
<tr>
<th>Front</th>
<th>Body</th>
<th>Mass(kg)</th>
<th>Moment of inertia( kg \cdot m^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chassis</td>
<td>1460.0</td>
<td>484, 2344, 2245</td>
<td></td>
</tr>
<tr>
<td>Rack</td>
<td>1.0</td>
<td>1.0, 1.0, 2.0</td>
<td></td>
</tr>
<tr>
<td>Lower control arm</td>
<td>3.0</td>
<td>2.0, 4.0, 2.0</td>
<td></td>
</tr>
<tr>
<td>Tie rod</td>
<td>5.0</td>
<td>4.0, 4.0, 4.0</td>
<td></td>
</tr>
<tr>
<td>Knuckle</td>
<td>4.0</td>
<td>3.0, 6.0, 3.0</td>
<td></td>
</tr>
<tr>
<td>strut</td>
<td>2.0</td>
<td>1.0, 1.0, 2.0</td>
<td></td>
</tr>
<tr>
<td>Spring constant</td>
<td></td>
<td>18639 N/m</td>
<td></td>
</tr>
<tr>
<td>Damping coefficient</td>
<td></td>
<td>1386 Ns/m</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rear</th>
<th>Body</th>
<th>Mass(kg)</th>
<th>Moment of inertia( kg \cdot m^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strut</td>
<td>2.0</td>
<td>2.0, 3.0, 2.0</td>
<td></td>
</tr>
<tr>
<td>Knuckle</td>
<td>3.0</td>
<td>3.0, 4.0, 3.0</td>
<td></td>
</tr>
<tr>
<td>Camber control arm</td>
<td>2.0</td>
<td>2.0, 3.0, 2.0</td>
<td></td>
</tr>
<tr>
<td>Toe control arm</td>
<td>2.0</td>
<td>1.0, 1.0, 2.0</td>
<td></td>
</tr>
<tr>
<td>Trail link</td>
<td>2.0</td>
<td>2.0, 2.0, 2.0</td>
<td></td>
</tr>
<tr>
<td>Spring constant</td>
<td></td>
<td>21582 N/m</td>
<td></td>
</tr>
<tr>
<td>Damping coefficient</td>
<td></td>
<td>1021 Ns/m</td>
<td></td>
</tr>
</tbody>
</table>

The sensitivity results are validated against these obtained from the finite difference method (FDM). Since the sensitivities are very small, the error tolerance of the integration must be maintained to be very small. Otherwise, accurate FDM results cannot be obtained. The error tolerance of $10^{-5}$ was used for this example. The analytic sensitivity and FDM sensitivity are shown to be identical in Fig. 5, which validates the purpose of this work. The sensitivity analysis was performed on an IBM compatible computer (266 Mhz) and took 10 min. This indicates that the sensitivity analysis of a fairly complicated system can be done quickly.

Fig. 4 Reaction force acting on the mounting point of the strut

Fig. 3 Step function

Fig. 5 Sensitivity of the reaction force on the welded area of the strut
5. Conclusions

A design sensitivity analysis method is proposed in this paper. Algorithms needed for sensitivity analysis is developed, and this makes possible to predict the parametric sensitivity. Sensitivities of the reaction force on welded chassis mounting area due to a damping coefficient change are obtained. The computing time indicated that sensitivity based design iteration of large scale mechanical systems is possible on the PC level computers with the proposed method.

Reference